Q9) (f) $\cot x - \tan x = \dfrac{4\cos^2 x - 2}{\sin 2x}$

$$LHS = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{\cos^2 x - (1-\cos^2 x)}{\sin x \cos x}$$

$$= \frac{2}{2}\left(\frac{2\cos^2 x - 1}{\sin x \cos x}\right) = \frac{4\cos^2 x - 2}{\sin 2x} = RHS.$$

Hence Proved.

Q9) (e) $\dfrac{\sin^2\theta - \tan\theta}{\cos^2\theta - \cot\theta} = \tan^2\theta.$

$$LHS = \frac{\sin^2\theta - \tan\theta}{\cos^2\theta - \cot\theta} = \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta - \frac{1}{\cos\theta}}{\cos\theta - \frac{1}{\sin\theta}}\right)$$

$$= \tan\theta\left(\frac{\sin\theta \cancel{\cos\theta - 1}}{\cancel{\sin\theta\cos\theta - 1}}\right) \cdot \times \frac{\sin\theta}{\cos\theta} = \tan\theta \times \tan\theta = \tan^2\theta$$

$= RHS.$

Hence Proved.

Q9 (d) $\dfrac{\sin\theta + \cos\theta\cot\theta}{\cos\theta\csc\theta} = \sec\theta.$

$$LHS. = \frac{\sin\theta + \cos\theta\cot\theta}{\cos\theta\csc\theta.} = \frac{\sin\theta + \cos\theta\frac{\cos\theta}{\sin\theta}}{\cos\theta\csc\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cancel{\sin\theta}\cos\theta\cancel{\csc\theta}} = \frac{1}{\cos\theta} = \sec\theta = RHS$$

Hence Proved.

Q9 (c) $\dfrac{1}{\sec\theta\tan\theta} = \csc\theta - \sin\theta$

$$LHS = \frac{1}{\sec\theta\tan\theta} = \frac{1}{\frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}} = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta}$$

$= \csc\theta - \sin\theta = RHS$

Hence proved.

Q9 (b) $\dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta} = 2\csc^2\theta$

$$LHS = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = \frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta} = \frac{2}{\sin^2\theta} = 2\csc^2\theta$$

$= RHS.$

Hence proved.

Q9 (a) $\frac{\csc^2\theta - 1}{\csc^2\theta} = \cos^2\theta$.

$$LHS = \frac{\csc^2\theta - 1}{\csc^2\theta} = 1 - \frac{1}{\csc^2\theta} = 1 - \sin^2\theta = \cos^2\theta = RHS.$$

Q8) $\cos\theta = -\frac{15}{17}$, $\theta$ lies in 2nd quadrant.

In quad. 2, $\sin\theta$ is +ve.

$H^2 = P^2 + B^2$

$17^2 = P^2 + (-15)^2$

$289 - 225 = P^2$

$\Rightarrow P^2 = 64$

$P = \pm 8$ ($P = -8$, neglected since $\sin\theta$ must be +ve)

$P = 8$.

$\sin\theta = \frac{8}{17}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{8}{15}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-\frac{16}{15}}{1-\frac{64}{225}} = \frac{-\frac{16}{15}}{\frac{161}{\cancel{225}\,15}} = -\frac{16 \times 15}{161}$$

$$= -\frac{240}{161}$$

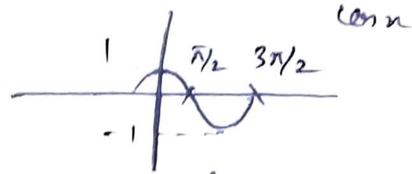
(b)

Q7) (a) $10\sin 4x\cos 4x = 5(2\sin 4x\cos 4x) = 5\sin 8x$.

(b) $4 - 8\cos^2 6x = 4(1 - 2\cos^2 6x) = -4(2\cos^2 6x - 1) = -4\cos 12x$

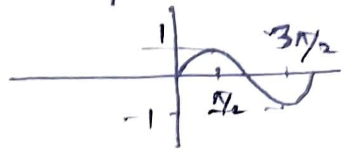
(c) $\frac{4\tan 3x}{1-\tan^2 3x} = 2\left(\frac{2\tan 3x}{1-\tan^2 3x}\right) = 2\tan 6x$

Q6 (a) $\cos\left(\frac{3\pi}{2} - x\right) = \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$




$= 0 \times \cos x + (-1) \times \sin x$

$= -\sin x.$




(b) $\tan x = 2$

$\tan\left(\frac{\pi}{3} - x\right) = \dfrac{\tan\frac{\pi}{3} - \tan x}{1 + \tan\frac{\pi}{3}\tan x}.$

$= \dfrac{\sqrt{3} - 2}{1 + 2\sqrt{3}} \times \dfrac{1 - 2\sqrt{3}}{1 - 2\sqrt{3}} = \dfrac{\sqrt{3} - 6 - 2 + 2\sqrt{3}}{1 - 12}$

$\tan\left(\frac{\pi}{3} - x\right) = \dfrac{8 - 3\sqrt{3}}{11}$

(c) $\sin\left(\frac{\pi}{3} - x\right) - \cos\left(\frac{\pi}{6} + x\right) = \left(\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right) - \left(\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x\right)$

$= \frac{\sqrt{3}}{2}\cos x - \cancel{\frac{1}{2}\sin x} - \cancel{\frac{\sqrt{3}}{2}\cos x} + \cancel{\frac{1}{2}\sin x} = 0.$

(d) $\cos 3x \cos 5x - \sin 3x \sin 5x$

Using $\cos(A+B) = \cos A\cos B - \sin A \sin B.$

$\therefore \cos 3x \cos 5x - \sin 3x \sin 5x = \cos 8x.$

(e) $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\pi - \frac{\pi}{12}\right) = \sin\pi\cos\frac{\pi}{12} - \cos\pi\sin\frac{\pi}{12}$

$= 0 - (-1)\sin\frac{\pi}{12} = \sin\frac{\pi}{12}$

Now $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$

Q6) (f) $\sin A = \frac{12}{13}$ in quad. 2.

$\sec B = \frac{5}{4}$ in quad 4.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

In quad 4, cos θ & sec θ are +ve

$\sec B = \frac{5}{4} \Rightarrow \cos B = \frac{4}{5}$

$H^2 = P^2 + B^2$

$(5)^2 = P^2 + 4^2$

$25 - 16 = P^2 \Rightarrow P = \pm\sqrt{9} = \pm 3$ $(P = +3)$ neglected.

$P = -3$

$\sin B = -\frac{3}{5}$

In quad 2, sin θ & csc θ are +ve.

$\sin A = \frac{12}{13}$.

$H^2 = P^2 + B^2$

$13^2 = 12^2 + B^2$

$B^2 = 169 - 144 = 25$

$B = \pm 5$ . $(B = 5$ neglected$)$

$B = -5$

$\cos A = \frac{-5}{13}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{-5}{13} \times \frac{4}{5} - \frac{12}{13} \times \left(-\frac{3}{5}\right) = \frac{-20 + 36}{65}$

$= \frac{16}{65}$

Q4 (a) $\sqrt{3}\tan x + 1 = 0$.

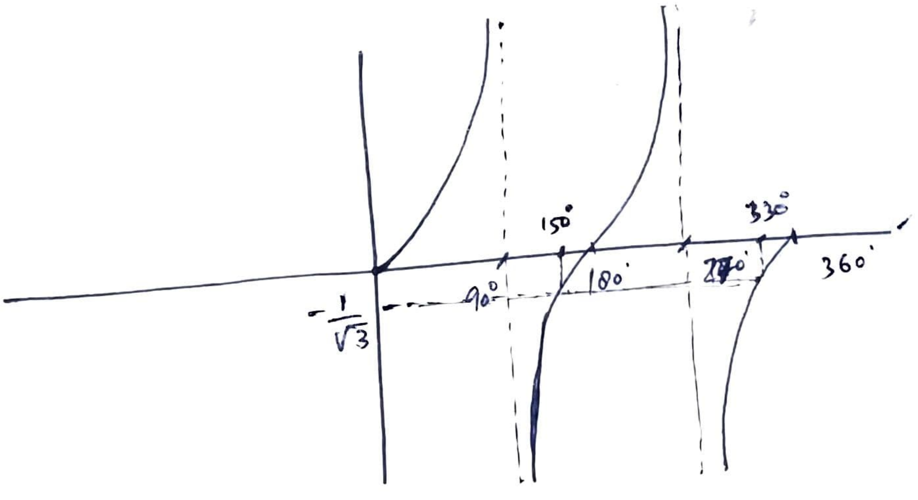
$\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(\pi - \pi/6\right) = \tan\frac{5\pi}{6}$

$x = n\pi + \frac{5\pi}{6}$

$x = \frac{5\pi}{6}, \ 11\pi/6$

$x = 150°, \ 330°$




tan x graph for $0° < x < 360°$

Q4) (c) $3\cos x = \cos x - 1$

$2\cos x = 1$

$\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{3}$

$x = \frac{\pi}{3}, \ \frac{5\pi}{3}$

$x = 60°, \ 300°$